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By Josef Taub

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Notation

α		enlargement factor for the weight of the seaplane.
α^{κ}		enlargement factor for the linear dimensions.
v_{crit}	(m/s)	critical starting speed.
v_{start}	(m/s)	starting speed.
v_{la}	(m/s)	landing speed.
β		angle at approach of seaplane on surface of water.
v	(m/s)	$= v_{la} \sin \beta$.
G	(kg)	gross weight.
M	($kg\ s^2/m$)	seaplane mass.
A		lift (with subscript l for air; without it, for water).
ρ	($kg\ s^2/m^2$)	density (with subscript l for air; without it, for water).
F	(m^2)	wing area.
c		coefficient of dynamic water lift.
c_a		coefficient of wing lift.

*"Beitrag zur Frage der Belastungsannahmen für den Landungsstoss von Seeflugzeugen." From Zeitschrift für Flugtechnik und Motorluftschiffahrt, July 28, 1931, pp. 433-442.

$\varphi = \sqrt{\frac{w}{1 + w}}$

$w = \frac{M_W}{M_r}$

$M_r = M \frac{i^2}{r^2 + i^2}$ (kg s^2/m) reduced seaplane mass with respect to the lateral axis.

i (m) radius of inertia of seaplane about the lateral axis.

r (m) vertical distance of force of impact from the lateral axis.

$\overline{M}_r$, $\overline{i}$, $\overline{r}$ the corresponding values with respect to the longitudinal seaplane axis.

$M_W = \frac{\pi\rho}{8} \frac{a^2\ b^2}{\sqrt{a^2 + b^2}} \left(1 - 0.425 \frac{a\ b}{a^2 + b^2}\right)$ (kg s^2/m) amount of water moved upon impact, according to Pabst.*

a (m) length of bottom impressed upon impact.

b (m) width of bottom impressed upon impact.

b_{st} (m) width of float at step.

$k = P/f$ (kg/m) elasticity of members stressed by impact.

f (m) deflection of members under impact stress.

P (kg) impact (force).

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*W. Pabst, Theory of the Landing Impact of Seaplanes. Z.F.M., Vol. 21, No. 9, May 14, 1930, pp. 217-226; and No. 16, p. 418. N.A.C.A. Technical Memorandum No. 580, 1930.
W. Pabst, Landing Impact of Seaplanes. Z.F.M., Vol. 22, No. 1, Jan. 14, 1931, pp. 13-28. N.A.C.A. Technical Memorandum No. 624, 1931.

n		impact factor.
p	(kg/m²)	wing area.
γ		keel angle (i.e., angle formed by intersection of bottom surfaces with one another).

Index α designates the values with respect to a seaplane of α-fold size. Index L and index R, respectively, denote the values according to Lanchester's and Rohrbach's enlargement method, respectively. Indexes c and d denote the respective values for single and double float arrangements. The line over the various letters refers to the case of one sided-landing of a twin-float seaplane (significance of $\overline{M}_r$, $\overline{i}$, $\overline{r}$ seen above).

The impact in dependence of size of the seaplane can be defined when the effect of the various quantities on the impact, and the changes in these quantities by enlargement of the seaplanes are known.

But it is an admitted fact that the seaplane can be enlarged in various ways, so that only the change of some quantities really is unambiguously affected by the enlargement, whereas the change of other quantities is also influenced by the method of enlargement. If the impact formula includes the latter quantities the impact factor cannot summarily be given as function of the seaplane size, but rather must embrace the type of enlargement as well.

Let the weight G of a large seaplane be α times greater than that of a small seaplane. By the enlargement the linear wing dimensions of the small seaplane are to be multiplied by α^{κ}.

In that case:

the wing area changes by	$\alpha^{2\kappa}$
" surface loading by	$\alpha^{1-2\kappa}$
" starting and landing speed by	$\alpha^{\frac{1}{2}-\kappa}$

The choice of κ defines the type of the wing enlargement.

The choice in float dimensions is contingent upon the conditions at starting, which include the attitude prior to and during planing.

Planing begins with the "hump speed," at which the float "rises to the step." In this attitude the static lift of the water is evanescently small; the total weight G is borne by the dynamic lift A of the water and by the lift of the aerodynamic forces A_l. As a result, the sum of both must be α times greater for the large than for the small seaplane.

Wagner* defines lift A at hump speed as

$$A = c\ \rho\ b_{st}^2\ \ v_{crit}^2 \tag{1}$$

and lift A_l as:

$$A_l = c_a\ \frac{\rho_l}{2}\ F\ v_{crit}^2 \tag{1a}$$

Now the ratio $\frac{v_{crit}}{v_{start}}$ is known to be fairly constant, so that it may be assumed that v_{crit}, like v_{start}, changes with $\alpha^{\frac{1}{2}-\kappa}$. F changes with $\alpha^{2\kappa}$, hence A_l changes with α. Consequently, the sum of A and A_l can only become α times greater than in the small seaplane when the dynamic lift of the water also increases with α. This happens when $c\ b_{st}^2$ changes with $\alpha^{2\kappa}$. Assuming that the coefficient of the dynamic lift of water c remains unchanged by the enlargement, the wetted length and width of the float must change with α^{κ}.

While accelerating to hump speed the gross weight of the seaplane is borne by the continuously decreasing water lift and by the continuously increasing dynamic lift of the water and of the air. A change in length and width with α^{κ} then is followed by an $\alpha^{1-2\kappa}$ fold greater immersion depth in the rest attitude than in the small seaplane. For $\kappa = \frac{1}{3}$ the immersion depth thus likewise changes with $\alpha^{\kappa} = \alpha^{\frac{1}{3}}$ and the form of the displaced water object by increased size is geometrically similar to that of

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*H. Wagner, The Landing of Seaplanes. Z.F.M., Vol. 22, No. 1, Jan. 14, 1931, pp. 1-8. N.A.C.A. Technical Memorandum No. 622, 1931.

original size. For $\frac{1}{3} < \kappa < \frac{1}{2}$ this is practically the only zone as far as κ is concerned; the immersion depth changes more slowly than width α^κ. The consequence is that the drag preceding hump speed becomes lower so that the large seaplane reaches the full planing attitude sooner than the small seaplane. According to this the stipulations of the attitude preceding hump speed are not contradictory to those of the planing attitude, conformably to which an enlargement of length and width of float with α^κ was not deemed necessary.

Now it becomes readily apparent that the enlargement of float length and width must as a rule be made with the same enlargement factor as the linear wing dimensions. The same prescribed type of wing enlargement is generally applied to the float also. In the following, we shall call term $\frac{G}{b_{st}^2}$ the float or boat loading corresponding in its analogy to the term for wing loading, hence the variation factors for the boat (float) loading and the wing loading are, in general, equal to one another.

This postulate, which, of course, is not to be taken as a strict design specification, presumes aerodynamically equivalent seaplanes, i.e., equivalent polars. In this sense, two seaplanes may be interpreted as aerodynamically equivalent even if one, to lower its get-away speed, resorts to a special arrangement (slotted wings, for instance), whose use, however, is confined to the immediate stage before get-away. It naturally includes also such cases in which one or the other of the comparative seaplanes is equipped with starting aids (towing planes, rockets, etc.) which merely serve to increase the traction without infringing on the get-away speed itself.

Consequently, factor κ is characteristic for the type of wing and float enlargement, and it defines, as a rule, not only the change in linear dimensions but also the change in starting and landing speed, as pointed out previously. Now, if the choice of κ decides the landing speed, the same can be substituted by a function of α and κ in the formula for the impact factor, which then includes, aside from constant quantities, α and κ only. Of course, the comparative seaplanes must be aerodynamically equivalent. They both must have devices for lowering the landing speed, such as slotted wings, if at all available. If the large seaplane is equipped with slotted wings and the small one is not, then of course, the landing speed is not merely dependent on α and κ, and in this case the landing speed must be included in the impact formula. But the linear dimensions of the floats change in this case also as those of the wing.

All these reflections are equally valid in cases of modification in the small seaplane (different surface loading), except that the modification factors are expressed as a power of the surface loading instead of α.

Recapitulating, it may be stated that the impact factor, under certain circumstances, can also be affected by the type of enlargement aside from the seaplane size and the landing speed. But the type of enlargement most generally defines the landing speed also, so that the knowledge of seaplane size and type of enlargement is not only necessary for interpreting the impact, but in most cases is even commensurate. By aerodynamically unequal enlargement the impact formula must embrace the landing speed or some other appropriate quantity.

In the following, we discuss these considerations on the basis of Pabst's and of Wagner's formulas:

Pabst derives

$$P = v \sqrt{k M_r \varphi} \tag{2}$$

as maximum impact for flat-bottom floats.

By α-fold enlargement of the seaplane the linear dimensions are to change with α^κ on the wing as well as on the floats conformably to our predicated assumption.

The accelerated water mass M_w depends on the impressed length and width of the bottom. Whereas the width changes with α^κ, the same cannot be summarily claimed for the length. By equal form and size of the waves, i.e., equal seaway, the length does not change. To allow for it would result in an abstruse dependence of M_w on the enlargement, which would further be augmented by the fact that distance r of the impact force from the C.G. of the seaplane would no longer change with α^κ by the enlargement and consequently, neither would M_r change with α, if the impacted length of the small seaplane is maintained.

For the sake of clarity, and since it is primarily a matter of explaining fundamental observations, it is assumed that the impacted bottom length changes like the other lengths. This means that greater waves (i.e., seaway) are presumed for the large seaplane than for the other. As a result the comparative planes are not wholly equivalent; the large seaplane has a stipulated greater seaworthiness than the small one.

This supposition leads, since M_w is proportional with a volume, to a water volume which increases with $\alpha^{3\kappa}$. Since M_r increases with α, w changes with $\alpha^{3\kappa-1}$, and the φ value of (2) for the large seaplane becomes

$$\varphi_\alpha = \sqrt{\frac{\alpha^{3\kappa-1}\, w}{1 + \alpha^{3\kappa-1}\, w}} = \alpha^{\frac{3\kappa-1}{2}} \sqrt{\frac{1 + w}{1 + \alpha^{3\kappa-1}\, w}}\ \varphi$$

(Index α refers to the large seaplane.)

The elasticity k in (2) may be expressed by

$$k = \frac{E}{\sigma}\frac{P}{s}$$

where E = elasticity modulus,
σ = stress upon impact,
and s = length of a member.

E and σ do not change by the enlargement, hence k changes, by similar total construction, like the P/s value. Value s changes - having the dimension of a length - with α^κ in first approximation. Thus

$$k_\alpha = \alpha^{-\kappa}\frac{P_\alpha}{P}\,k.$$

The landing speed $v_{l\,a}$ and with it the value v changes with $\alpha^{\frac{1}{2}-\kappa}$ according to our considerations, stated at the beginning of the above remarks.

Now the force of impact of the large seaplane is

$$P_\alpha = \alpha^{\frac{1}{2}-\kappa} v \sqrt{\alpha^{-\kappa} \frac{p_\alpha}{p} k \alpha M_r \alpha^{\frac{3\kappa-1}{2}}} \sqrt{\frac{1+w}{1+\alpha^{3\kappa-1} w}} \varphi =$$

$$= \alpha \frac{1+w}{1+\alpha^{3\kappa-1} w} P \qquad (3)$$

The impact factor for the large seaplane is revealed by the ratio of impact force to total weight:

$$n_\alpha = \frac{P_\alpha}{G_\alpha} = \frac{1+w}{1+\alpha^{3\kappa-1} w} n \qquad (4)$$

and, if this seaplane is equipped with some device for changing its landing speed or the value v with respect to the corresponding value of the small seaplane, the impact force is

$$P_\alpha = \alpha^{2\kappa} \frac{1+w}{1+\alpha^{3\kappa-1} w} \left(\frac{v_\alpha}{v}\right)^2 P \qquad (3a)$$

where v_α denotes the v value for the enlarged type.

In this case the impact factor is

$$n_\alpha = \alpha^{2\kappa-1} \frac{1+w}{1+\alpha^{3\kappa-1} w} \left(\frac{v_\alpha}{v}\right)^2 n \qquad (4a)$$

The κ value, which is difficult to define in practice, can now be substituted by a function of the surface loadings and of enlargement α. Of course, κ could equally be expressed by the landing speed and α. But in the general case (4a) there is no connection between κ and v, hence our choice for the former possibility.

For wing loading, we have

$$p_\alpha = p \, \alpha^{1-2\kappa} \qquad (5)$$

and consequently,

$$\alpha^{2\kappa-1} = \frac{p}{p_\alpha} \qquad (6)$$

In addition, equation (5) yields

$$\kappa = \frac{1}{2}\left(1 - \frac{\ln \frac{p_\alpha}{p}}{\ln \alpha}\right) \tag{7}$$

and

$$\alpha^{3\kappa - 1} = \sqrt{\left(\frac{p}{p_\alpha}\right)^3 \alpha} \tag{8}$$

With (6) and (8) we find from (4) and (4a):

$$\boxed{n_\alpha = \Phi\, n} \tag{9}$$

and

$$\boxed{n_\alpha = \left(\frac{v_\alpha}{v}\right)^2 \frac{p}{p_\alpha}\, \Phi\, n} \tag{9a}$$

where

$$\Phi = \frac{1 + w}{1 + w \sqrt{\alpha \left(\frac{p}{p_\alpha}\right)^3}} \tag{10}$$

In these formulas, n, p, v, and w are the values for the small seaplane, as given or defined once for all by experiment. To define the impact factor for a stated seaplane thus requires only the enlargement factor α and the surface loading p. For seaplanes whose lift coefficient at landing differs from that of the original, it further requires the landing speed (v_α, respectively). This, however, is practically confined to seaplanes equipped with special devices for lowering the landing speed.

The κ value characterizing the type of enlargement and the wing loading p_α can now be chosen differently. The criterion, in the first place, is the wing stress in flight and during landing. As the loading increases, the linear dimensions of the wing become smaller and consequently its weight, in so far as this is conditioned by stresses in flight.

On the other hand, the higher wing loading is given a limit by the higher landing speed involved with it.

Outside of the fact that through it the landing phase may become of marked significance for the dimensioning of the wing, particularly in large seaplanes with whole or partial load distribution over the span, it results in abnormally weighty floats or boats.

According to past experiences, the lower limit of wing load enlargement appears to follow Lanchester's* - the upper, Rohrbach's** type of enlargement.

According to Lanchester the enlargemont is $\kappa = \frac{1}{2}$, hence,

the lengths change with $\alpha^{1/2}$
" surfaces, with α
" surface loadings, with $\alpha^0 = 1$
and " starting and landing speeds, with $\alpha^0 = 1$.

According to Rohrbach,

$\kappa = 1/3$ and
the lengths change with $\alpha^{1/3}$
" surfaces, with $\alpha^{2/3}$
" surface loadings, with $\alpha^{1/3}$
and " landing speeds, with $\alpha^{1/6}$

Lanchester's type, according to (9) and (10), yields:

$$n_{\alpha_L} = \frac{1 + w}{1 + w\alpha^{1/2}} n \tag{11}$$

and Rohrbach's

$$n_{\alpha_R} = n \tag{12}$$

and according to (9a) and (10):

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*F. W. Lanchester, The Development of the Military Airplane; the Question of the Size. Engineering, March 3, 1916, p. 213.
**A. Rohrbach, The Enlargement of Airplanes. Jahrbuch der W.G.L., 1922, p. 37.

$$n_{\alpha_L} = \left(\frac{v_\alpha}{v}\right)^2 \frac{1 + w}{1 + w\alpha^{1/2}} n \tag{11a}$$

$$n_{\alpha_R} = \left(\frac{v_\alpha}{v}\right)^2 \frac{n}{\alpha^{1/3}} \tag{12a}$$

Assume the Heinkel HE8 used by Pabst in his studies, as the small seaplane, and for which he obtained (Z.F.M., Vol. 21, 1930, p. 223):

$$w = 0.15.$$

Accordingly, (11) would reveal:

$$n_{\alpha_L} = \frac{1.15}{1 + 0.15\ \alpha^{1/2}} n$$

The $\frac{n_{\alpha_L}}{n}$ and the $\frac{n_{\alpha_R}}{n}$ values were calculated under this assumption for various α and plotted in Figure 1.

Based upon Pabst's formula for flat-bottom floats, it is readily apparent that as far as reduced landing impact is concerned - as expected - Lanchester's type is preferable to Rohrbach's.

Pabst's formula for sharp-keeled bottoms with constant "V" along the forebody gives the force of impact as

$$P = \tan\frac{\gamma}{2}\, v^2\, \frac{c_0\ \rho\ a^2}{(1 + w)^3} \tag{13}$$

with

$$c_0 = 0.786\ e^{-0.227\ a/b}$$

By α-fold enlargement, where the linear dimensions are again to change with α^k we obtain, by similar considerations as heretofore and with the further notation

$$c_1 = c_0\ \rho\ \tan\frac{\gamma}{2}$$

$$P_\alpha = c_1\ \alpha^{1-2K}\ v^2\ \frac{\alpha^{2K}\ a^2}{(1 + \alpha^{3K-1}\ w)^3} = \alpha\ \left(\frac{1 + w}{1 + \alpha^{3K-1}\ w}\right)^3\ P \qquad (14)$$

and

$$n_\alpha = \frac{P_\alpha}{G_\alpha} = \left(\frac{1 + w}{1 + \alpha^{3K-1}\ w}\right)^3\ n \qquad (15)$$

or, if different lift coefficients are decisive for the landing speed of the compared seaplanes, the more general formulas:

$$P_\alpha = \left(\frac{v_\alpha}{v}\right)^2\ \alpha^{2K}\ \left(\frac{1 + w}{1 + \alpha^{3K-1}\ w}\right)^3\ P \qquad (14a)$$

and

$$n_\alpha = \left(\frac{v_\alpha}{v}\right)^2\ \alpha^{2K-1}\ \left(\frac{1 + w}{1 + \alpha^{3K-1}\ w}\right)^3\ n \qquad (15a)$$

Expressing K again by $\frac{p}{p_\alpha}$, formulas (15) and (15a) reveal:

$$\boxed{n_\alpha = \Phi^3\ n} \qquad (16)$$

and

$$\boxed{n_\alpha = \left(\frac{v_\alpha}{v}\right)^2\ \frac{p}{p_\alpha}\ \Phi^3\ n} \qquad (16a)$$

where Φ is defined by (10).

The impact factor increases in this case also when the surface loading increases, according to (16), (16a), and (10). Consequently, the wing weight resulting from the landing case will under certain circumstances also increase with the surface loading, so that the conditions of landing impose a limitation in respect to the increase in favorable surface loading. The appropriate loading depends on the shape of the wing, load distribution, etc. Its selection is a problem by itself and outside the scope of this paper.

We again confine ourselves to the study of the conditions in the cases of Lanchester's and Rohrbach's type of enlargement. According to (16) and (10), Lanchester's method yields

$$n_{\alpha_L} = \left(\frac{1 + w}{1 + w\ \alpha^{1/2}}\right)^3 n \tag{17}$$

and Rohrbach's method yields

$$n_{\alpha_R} = n \tag{18}$$

or, according to (16a) and (10), the general formula

$$n_{\alpha_L} = \left(\frac{v_\alpha}{v}\right)^2 \left(\frac{1 + w}{1 + w\ \alpha^{1/2}}\right)^3 n \tag{17a}$$

and

$$n_{\alpha_R} = \left(\frac{v_\alpha}{v}\right)^2 \frac{n}{\alpha^{1/3}} \tag{18a}$$

Again we assume $w = 0.15$ and compute $\frac{n_{\alpha_L}}{n}$ and $\frac{n_{\alpha_R}}{n}$ according to (17) and (18), as illustrated in Figure 2.

Here Lanchester's type corresponds to a much quicker drop in load factor than for the same type with flat bottoms.

Wagner's formula for straight "V" bottoms and constant keel angle along the forebody gives the impact force as

$$P = c_2\ v^2 \sqrt{a\ M_r}\text{*} \tag{19}$$

where

$$c_2 = 0.835\sqrt{\rho}\left\{\tan\frac{\gamma}{2} - \left[\frac{1}{\pi} + \frac{2}{\pi^2}\left(0.15 + \ln\frac{\pi}{2}\tan\frac{\gamma}{2}\right)\right]\right\}$$

By α-fold enlargement of the original size, where the linear dimensions again are to change with α^κ, we have

$$P_\alpha = c_2\ \alpha^{1-2\kappa}\ v^2 \sqrt{\alpha^\kappa\ a\ \alpha\ M_r} =$$

$$= \alpha^{\frac{3}{2}(1-\kappa)}\ P \tag{20}$$

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*Wagner's symbols changed to conform to our notations.

and
$$n_\alpha = \frac{P_\alpha}{G_\alpha} = \alpha^{\frac{1}{2}(1-3\kappa)}\ n \tag{21}$$

or, if different lift coefficients are decisive for the landing speed of the compared sizes, the more general formulas:

$$P_\alpha = \left(\frac{v_\alpha}{v}\right)^2 \alpha^{\frac{1}{2}(\kappa+1)}\ P \tag{20a}$$

and
$$n_\alpha = \left(\frac{v_\alpha}{v}\right)^2 \alpha^{\frac{1}{2}(\kappa-1)}\ n \tag{21a}$$

Again we introduce the surface loadings so that (21) and (21a), respectively, yield:

$$\boxed{n_\alpha = \frac{1}{\sqrt[4]{\left(\frac{p}{p_\alpha}\right)^3 \alpha}}\ n} \tag{22}$$

and

$$\boxed{n_\alpha = \left(\frac{v_\alpha}{v}\right)^2 \frac{1}{\sqrt[4]{\alpha\,\frac{p_\alpha}{p}}}\ n} \tag{22a}$$

The impact factor increases with the surface loading according to (22) so that the statements made relative to (16) are applicable to this case also.

For Lanchester's enlargement, (22) yields

$$n_{\alpha_L} = \frac{n}{\alpha^{1/4}} \tag{23}$$

and for Rohrbach's enlargement, $n_{\alpha_R} = n$ (24)

or, conformably to (22a), the general formulas,

$$n_{\alpha_L} = \left(\frac{v_\alpha}{v}\right)^2 \frac{n}{\alpha^{1/4}} \tag{23a}$$

$$n_{\alpha_R} = \left(\frac{v_\alpha}{v}\right)^2 \frac{n}{\alpha^{1/3}} \tag{24a}$$

In Figure 3 the $\frac{n_{\alpha_R}}{n}$ and $\frac{n_{\alpha_L}}{n}$ values were computed from (23) and (24) for various α.

As previously pointed out, the seaway in the impact formulas is characterized by the length a. Hitherto we stipulated that length a increases with the size of the seaplane and in the same measure as the other linear dimensions, which was very fortunate when applied to Pabst's formula. But this reduction is superfluous, or nearly so, in many cases, with Wagner's formula, without detracting from the simplicity of the results. Thus, in order to be able to follow the effect of the seaway in one case at least, we apply Wagner's formula to a length a, which is not changed by the enlargement. We solely consider the case of centric impact, i.e., the cases where $r = 0$ and consequently, $M_r = M$. In this case the impact formula is not modified save for the speed and the mass M_r, although the change in the latter is independent of the type of enlargement. Now the general formula for the load factor reads as

$$n_\alpha = \left(\frac{v_\alpha}{v}\right)^2 \frac{n}{\alpha^{1/2}} \tag{25}$$

with Lanchester's type as

$$n_{\alpha_L} = \frac{n}{\alpha^{1/2}} \tag{26}$$

and Rohrbach's,

$$n_{\alpha_R} = \frac{n}{\alpha^{1/6}} \tag{27)*$$

A comparison of (26) and (27) with (23) and (24) reveals, as expected, a markedly faster decrease in load factors as the size increases than in the previous assumption, when stipulating the same seaway.

Lastly, we add, that (25) with $\alpha = 1$ represents the impact and the load factor, respectively, for the case when changing from the original size to another of the same size but different landing speed. Pabst's formulas

*Since by the Rohrbach method the impacted width of the bottom pertaining to maximum impact increases more rapidly than the full width of the float bottom, (27) remains valid only so long as its corresponding impacted bottom width does not exceed the full width of the float bottom.

do not bring out the effect of this change quite so clearly, because a change in landing speed is followed, according to our reflections, by a change in float width b_{st}, whereas the impacted length a remains the same conformably to the unchanged seaway; the c_0 and w values in the impact formula change, as a result, in complicated manner.

In accord with the design specifications in force at the present time, the sum of the impact forces on both floats of a twin-float seaplane must be equated with the impact force on the hull of a flying boat of the same weight and landing speed, although the actual conditions are quite often to the contrary.

Let subscript e represent the respective quantities of the flying boat and d those of the twin-float seaplane, so that the dynamic lift at critical starting speed, according to (1) becomes:

$$A_e = c\ \rho\ b_{st_e}^2\ v_{crit_e}^2 \tag{28}$$

$$A_d = 2\ c\ \rho\ b_{st_d}^2\ v_{crit_d}^2 \tag{28a}$$

By equal weight and equal starting speed $A_e = A_d$ and approximately $v_{crit_e} = v_{crit_d}$, so that according to (28) and (28a)

$$b_{st_d} = \frac{1}{\sqrt{2}}\ b_{st_e} \tag{29}$$

When changing from the single to the twin-float type the step width must be reduced according to Lanchester's method, i.e., the float loading must be retained.

Now the impact forces can be computed for both arrangements from the divers impact formulas with, of course, the same seaway. The sea is expressed by the impacted bottom length a, and must not be changed in the two arrangements.

Assuming the distance of the step from the C.G. in single and twin-float types to be the same* distance r and consequently M_r will be the same for both types.

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*This assumption - without which, it would be impossible to make any general study - is probably always complied with as to height in the case of equal mass distribution.

Since, according to the preceding statement, the impact factors increase by a reduction conformably to Lanchester's method, and since, in addition, the impacted length is not reduced in the present case, but remains constant, it is to be expected that the impact on both floats of a twin-float type is probably higher than that on the corresponding flying boat. This inference is fundamentally confirmed by Pabst's and by Wagner's impact formulas, although the results, individually, are open to suspicion.

This brings us to Pabst's formula for flat bottoms. We begin by computing φ_e and φ_d for various practically possible values of a/b:

For the single float type:

$$M_{w_e} = \frac{\pi\rho}{8} \frac{a^2 \, b_e^2}{\sqrt{a^2 + b_e^2}} \left(1 - 0.425 \frac{a \, b_e}{a^2 + b_e^2}\right).$$

For one float of a twin-float type, we have, with regards (29)

$$M_{w_d} = \frac{\pi\rho}{8} \frac{a^2 \, b_e^2}{\sqrt{2}\sqrt{2 \, a^2 + b_e^2}} \left(1 - 0.425 \sqrt{2} \frac{a \, b_e}{2 \, a^2 + b_e^2}\right) =$$

$$= \frac{\pi\rho}{8} \frac{a^2 \, b_e^2}{\sqrt{2 \, a^2 + b_e^2}} \left(0.707 - 0.425 \frac{a \, b_e}{2 \, a^2 + b_e^2}\right),$$

hence for

$$\underline{\frac{a}{b_e} = \frac{1}{2}}$$

$$M_{w_e} = \frac{\pi\rho}{8} \frac{4 \, a^4}{\sqrt{5} \, a} \left(1 - 0.425 \frac{2 \, a^2}{5 \, a^2}\right) = \frac{\pi\rho}{8} 1.48 \, a^3$$

$$M_{w_d} = \frac{\pi\rho}{8} \frac{4 \, a^4}{\sqrt{6} \, a} \left(0.707 - 0.425 \frac{2 \, a^2}{6 \, a^2}\right) =$$

$$= \frac{\pi\rho}{8} 0.92 \, a^3 = 0.62 \, M_{w_e},$$

for

$$\frac{a}{b_e} = 1:$$

$$M_{w_e} = \frac{\pi\rho}{8} \frac{a^4}{\sqrt{2}\ a} \left(1 - 0.425 \frac{a^2}{2\ a^2}\right) = \frac{\pi\rho}{8}\ 0.555\ a^3$$

$$M_{w_d} = \frac{\pi\rho}{8} \frac{a^4}{\sqrt{3}\ a} \left(0.707 - 0.425 \frac{a^2}{3\ a^2}\right) =$$

$$= \frac{\pi\rho}{8}\ 0.326\ a^3 = 0.59\ M_{w_e},$$

for

$$\frac{a}{b_e} = 4:$$

$$M_{w_e} = \frac{\pi\rho}{8} \frac{a^4}{4\sqrt{17}\ a} \left(1 - 0.425 \frac{4\ a^2}{17\ a^2}\right) = \frac{\pi\rho}{8}\ 0.055\ a^3$$

$$M_{w_d} = \frac{\pi\rho}{8} \frac{a^4}{4\sqrt{33}\ a} \left(0.707 - 0.425 \frac{4\ a^2}{33\ a^2}\right) =$$

$$= \frac{\pi\rho}{8}\ 0.029\ a^3 = 0.53\ M_{w_e}.$$

We generally put:

$$M_{w_d} = M_{w_e}.$$

Furthermore,

$$w_d = \frac{2\ M_{w_d}}{M_r} \quad \text{and} \quad w_e = \frac{M_{w_e}}{M_r} = \frac{w_d}{2\nu}.$$

Consequently,

$$\varphi_d = \sqrt{\frac{w_d}{1 + w_d}} = \sqrt{\frac{2\nu + w_d}{1 + w_d}}\ \varphi_e \tag{30}$$

Regarding the change in elasticity k while changing from single to twin-float type, various assumptions are possible. We suppose that both types merely differ in the number of their floats from one another. Assuming equal stresses, the first approximation then yields equal deflections f. The value $k = P/f$ then is proportional to the impact forces, i.e.:

$$k_d = k_e \frac{P_d}{P_e}. \tag{31}$$

Now (2) reveals the impact P_d on one float of the twin-float seaplane at

$$P_d = v \sqrt{k_d \frac{M_r}{2}} \varphi_d = \frac{\nu + \frac{w_d}{2}}{1 + w_d} P_e \tag{32}$$

Expressing ν by its mean value of about 0.57, we have:

$$P_d = \frac{0.57 + 0.50 \, w_d}{1 + w_d} P_e \tag{33}$$

The magnitude of w_d depends on the size of the seaplane and the type of enlargement. For the Heinkel HE8, Pabst's method yields $w_d = 0.15$. Enlarging this plane by the Rohrbach method, leaves w_d unchanged, and

$$P_d = 0.56 \, P_e,$$

or, in other words, the total force of impact on the seaplane is 12% higher than on the equivalent flying boat.

According to Lanchester's method w_d increases with the root of the enlargement factor; the ratio P_d/P_e decreases and approaches the limit value 0.5.

It is only when the surface loading increases more rapidly than by the Rohrbach method that more marked increases in impact force than 12% would come into question while changing from flying boat to seaplane.

Thus, summarizing, it may be stated that the change from single to twin-float type - by flat bottom - under the stipulated assumptions is accompanied by increased impact force, though generally not important enough to be absolutely essential of inclusion in the specifications.

With respect to Pabst's formula for sharp "V" bottoms, formula (13) yields for the single float type:

$$P_e = \tan \frac{\gamma}{2} v^2 \frac{c_{o_e} \rho \, a^2}{(1 + w_e)^3} \tag{34}$$

where

$$c_{o_e} = 0.786\ e^{-0.227\ a/b_e} \tag{35}$$

and formula (29) yields for one float of a twin-float type

$$P_d = \tan \frac{\gamma}{2}\ v^2\ \frac{c_{o_d}\ \rho\ a^2}{(1 + w_d)^3} \tag{36}$$

where

$$c_{o_d} = 0.786\ e^{-0.227\ \sqrt{2}\ a/b_e} \tag{37}$$

Conformably to (35) and (37)

$$c_{o_d} = e^{-0.094\ a/b_e}\ c_{o_e} \tag{38}$$

Inserting (38) and (36) and again replacing w_e by $w_d/2\nu$, we find through (34) that

$$P_d = e^{-0.094\ a/b_e} \left(\frac{1 + \frac{w_d}{2\nu}}{1 + w_d}\right)^3 P_e \tag{39}$$

This formula is to be evaluated for various a/b_e values. First, we again use Pabst's $w_d = 0.15$, as defined for the Heinkel HE8. The values $\nu = M_{w_d}/M_{w_e}$ have been previously computed. We find

for $\underline{\frac{a}{b_e} = \frac{1}{2}}$: $\nu = 0.62$

$$P_d = e^{-0.047} \left(\frac{1 + \frac{0.15}{2 \times 0.62}}{1.15}\right)^3 P_e = 0.88\ P_e,$$

for $\underline{\frac{a}{b_e} = 1}$: $\nu = 0.59$

$$P_d = e^{-0.094} \left(\frac{1 + \frac{0.15}{2 \times 0.59}}{1.15}\right)^3 P_e = 0.85\ P_e,$$

for $\frac{a}{b_e} = 4$: $\qquad \nu = 0.53$

$$P_d = e^{-0.376} \left(\frac{1 + \frac{0.15}{2 \times 0.53}}{1.15}\right)^3 = 0.67\ P_e.$$

These values are applicable to larger types also, providing the enlargement follows Rohrbach's method. If, on the other hand, it follows Lanchester's method, then we have, for example: For $\alpha = 20$ because

$$w_d = \sqrt{20} \times 0.15 = 0.67$$

for $\frac{a}{b_e} = \frac{1}{2}$: $\qquad \nu = 0.62$

$$P_d = e^{-0.047} \left(\frac{1 + \frac{0.67}{2 \times 0.62}}{1.67}\right)^3 P_e = 0.77\ P_e,$$

for $\frac{a}{b_e} = 1$: $\qquad \nu = 0.59$

$$P_d = e^{-0.094} \left(\frac{1 + \frac{0.67}{2 \times 0.59}}{1.67}\right)^3 P_e = 0.75\ P_e,$$

for $\frac{a}{b_e} = 4$: $\qquad \nu = 0.53$

$$P_d = e^{-0.376} \left(\frac{1 + \frac{0.67}{2 \times 0.53}}{1.67}\right)^3 P_e = 0.64\ P_e.$$

It is seen that the ratio P_d/P_e decreases also with increasing a/b_e.

A general formula is obtained when substituting mean values (say $\nu = 0.57$ for $\frac{a}{b_e} = 1.5$) in (39) for ν and $\frac{a}{b_e}$, and replacing w_d by $w \sqrt{\left(\frac{p}{p_\alpha}\right)^3 \alpha}$, as explained previously. This would result in

$$P_d = 0.87 \left[\frac{1 + 0.88\, w \sqrt{\left(\frac{p}{p_\alpha}\right)^3 \alpha}}{1 + w \sqrt{\left(\frac{p}{p_\alpha}\right)^3 \alpha}} \right]^3 P_e \tag{39a}$$

Since P_d/P_e decreases by increasing surface loading, and we do not intend to presume for practical purposes a faster increase than according to Rohrbach's method, the defined P_d/P_e value may be considered as maximum for this type of enlargement.

As a result, $P_d = 0.75\ P_e$ (40)

may therefore be considered as a very rough average of Pabst's formula.

According to this the force of impact for the twin-float type with "V" bottom would be about 50% higher than for an equivalent single float, or flying boat type.

The application of (19) to one float of the twin-float type, according to Wagner's formula for "V" bottoms yields:

$$P_d = c_2\, v^2 \sqrt{a \frac{M_r}{2}} = \frac{1}{\sqrt{2}}\, P_e = 0.71\ P_e \tag{41}$$

Conformably to this formula the force on impact for the twin-float type would exceed that of the single float or flying boat type by 42%.

Even though the numerical results of our investigation fluctuate considerably, it is nevertheless apparent that the change from single to twin-float type with "V" bottom is accompanied by a material increase in impact. The respective specifications should be modified for "V" bottom. According to past experiments the ratio of force of impact in the twin-float type to that of the single-float type with sharp "V" bottoms seems to be about 1.5:1. Since this ratio for flat bottoms does not appear to deviate very much from 1:1, smaller keel angles should be given a value between 1:1 and 1:1.5.

A feature of incidental interest resulting from this investigation is that whereas, by flat bottoms the total impact is scarcely reduced during the change from twin-float to single-float type, much may be gained by this measure with "V" bottoms.

The structural specifications in force at the present date stipulate one-half the impact force by one-sided landing of a twin-float seaplane of that prescribed for simultaneous landing of both floats.

Since an eccentric impact depends on the reduced mass and this is contingent upon the mass inertia moment and the eccentricity of the shock, and lastly, since the latter varies appreciably in the different seaplanes, it is questionable whether such sweeping specification really does justice to practical conditions. Let r be the eccentricity of the impact with respect to the longitudinal axis of the seaplane (fig. 4) and i the inertia radius. Then the reduced mass of seaplane is

$$\overline{M}_r = M \frac{1}{1 + \frac{\overline{r}^2}{\overline{i}^2}} = \tau M \tag{42}$$

(In this inquiry it is assumed that there is no eccentricity with respect to the lateral axis.)

The size of $\overline{r}^2/\overline{i}^2$ governs M_r and consequently, the eccentric impact $\overline{P}$. The greater the eccentricity and the smaller the inertia moment, the greater is $\overline{r}^2/\overline{i}^2$. The inertia moment is the greater the more the mass of the seaplane is decentralized.

The author investigated the practically probable conditions for several arrangements* (loads centrically arranged, loads concentric at isolated parts of the wing, loads evenly distributed along different parts of the span), and found that

$$0.1 < \frac{\overline{r}^2}{\overline{i}^2} < 2.8.$$

The $\frac{\overline{r}^2}{\overline{i}^2} = 2.8$ is obtained by central loading and widely spaced floats.

The practical limiting values of $\overline{r}^2/\overline{i}^2$ yield approximately $0.25 < \tau < 0.90$.

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*To be published later.

In the following we compare eccentric impact $\overline{P}$ with impact P_d for a float of a twin-float seaplane by centric impact, based upon Pabst's and Wagner's formulas.

In Pabst's formula for flat bottoms, we denote the quantity with respect to the eccentric impact by an overline, so that (2) reveals

$$\overline{P} = v \sqrt{\overline{k}\ \overline{M}_r}\ \overline{\varphi} \tag{43}$$

Now $\overline{M}_w = M_{w_d}$, hence

$$\overline{w} = \frac{M_{w_d}}{\overline{M}_r} = \frac{M_{w_d}}{M_r} = \frac{w_d}{2\tau}$$

further,

$$\overline{\varphi} = \sqrt{\frac{\overline{w}}{1 + \overline{w}}} = \sqrt{\frac{1}{2\ \tau}} \sqrt{\frac{1 + w_d}{1 + \frac{w_d}{2\tau}}}\ \varphi_d$$

Quantity $\overline{k}$ may be equated to k_d in first approximation, by assuming the deflections for one-sided and centric impact to be proportional to the corresponding impact forces per float.

Then (43) yields:

$$\overline{P} = v \sqrt{k_d \frac{M}{2} \varphi_d} \sqrt{\frac{1 + w_d}{1 + \frac{w_d}{2\tau}}} \sqrt{\frac{1 + w_d}{1 + \frac{w_d}{2\tau}}}\ P_d \tag{44}$$

The specifications in force to-day prescribe $\overline{P} = P_d$. According to (44) this value is only obtained when $\tau = 0.5$, that is, when $\overline{r}^2/\overline{i}^2 = 1$.

In order to obtain some information concerning the magnitude of $\overline{P}$ under other conditions, we evaluate (44) for the limiting value of τ and for $w_d = 0.15$ and, corresponding to a 20-fold Lanchester enlargement, for $w_d = \sqrt{20} \times 0.15 = 0.67$.

For $w_d = 0.15$, we find:

$$\bar{P} = \sqrt{\frac{1 + 0.15}{1 + \frac{0.15}{2 \times 0.25}}} P_d = 0.94 \, P_d$$

and

$$\bar{P} = \sqrt{\frac{1 + 0.15}{1 + \frac{0.15}{2 \times 0.90}}} P_d = 1.03 \, P_d;$$

for $w_d = 0.67$

$$\bar{P} = \sqrt{\frac{1 + 0.67}{1 + \frac{0.67}{2 \times 0.25}}} P_d = 0.85 \, P_d$$

and

$$\bar{P} = \sqrt{\frac{1 + 0.67}{1 + \frac{0.67}{2 \times 0.90}}} P_d = 1.11 \, P_d.$$

It is seen that a 20-fold Lanchester enlargement occasions a 15% discrepancy with the specifications in the extreme case.* Since structural specifications, like this investigation, can no more than express rough approximations of actual conditions, the interpretations heretofore concerning one-sided impact for flat floats may be considered as correct according to the present stage of development. Of course, for very large seaplanes with comparatively light surface loading and an $\bar{r}^2/\bar{i}^2$ value appreciably deviating from 1.0, it is advisable to define the amount of one-sided impact more accurately.

With Pabst's formula for "V" bottoms, the problem can be simplified by assuming the direction of impact into the symmetrical plane of the float notwithstanding the one-sided landing. A case in point is when the seaplane, while landing, evinces neither an inclined position nor a velocity component outside of its plane of symmetry, and

*By an arbitrary Rohrbach enlargement the values $w_d = 0.15$ are valid.

when the one-sidedness of the landing impact is solely due to the fact that one float strikes the crest of a wave. In such cases we have, according to (13)

$$\overline{P} = \tan \frac{\gamma}{2} v^2 \frac{c_0 \rho a^2}{(1 + \overline{w})^3} \tag{45}$$

Now $\overline{c}_0 = c_{o_d}$ and, as for the case of flat bottoms

$$\overline{w} = \frac{w_d}{2\tau}$$

Therefore,

$$\overline{P} = \left(\frac{1 + w_d}{1 + \frac{w_d}{2\tau}}\right)^3 P_d \tag{46}$$

The $\overline{P} = P_d$ value is again reached, according to the specifications in force, only when $\tau = 0.5$, that is, for $\overline{r}^2/\overline{i}^2 = 1$, as in the case of flat bottoms.

Now we again interpret (46) for the practical limiting values of τ and for $w_d = 0.15$ and, corresponding to a 20-fold Lanchester enlargement, for $w_d = 0.67$:

For $w_d = 0.15$, we find:

$$\overline{P} = \left(\frac{1 + 0.15}{1 + \frac{0.15}{2 \times 0.25}}\right)^3 P_d = 0.70 \ P_d$$

and

$$\overline{P} = \left(\frac{1 + 0.15}{1 + \frac{0.15}{2 \times 0.90}}\right)^3 P_d = 1.20 \ P_d;$$

for $w_d = 0.67$, we have:

$$\overline{P} = \left(\frac{1 + 0.67}{1 + \frac{0.67}{2 \times 0.25}}\right)^3 P_d = 0.36 \ P_d$$

and

$$P = \left(\frac{1 + 0.67}{1 + \frac{0.67}{2 \times 0.90}}\right)^3 P_d = 1.81\ P_d.$$

Here marked discrepancies (over or under) with the existing design specifications may occur as the size of the seaplane increases. (For example, by a 20-fold Lanchester enlargement of the original type, the excess amounts to 81%!) Consequently, - admitting the validity of Pabst's formula - it is not correct to prescribe a constant ratio of the impact forces occurring during centric and one-sided landing on one float (hitherto 1), but rather to express the load factor for the one-sided impact as function of the enlargement as well as of its type, of the inertia moment and of the eccentricity. Such a formula can be set up summarily by combining (46) with (16) and (16a), respectively, by putting

$$\bar{n} = \frac{\bar{P}}{G}$$

$$n_d = \frac{P_d}{G}$$

and

$$w_{d_\alpha} = w_d \sqrt{\left(\frac{p}{p_\alpha}\right)^3 \alpha}$$

into (46). Then we obtain:

$$\bar{n}_\alpha = \left[\frac{1 + w_d}{1 + \frac{w_d}{2\tau}\sqrt{\left(\frac{p}{p_\alpha}\right)^3 \alpha}}\right]^3 n_d \qquad (47)$$

and the more general formula

$$\bar{n}_\alpha = \left(\frac{v_\alpha}{v}\right)^2 \left(\frac{p}{p_\alpha}\right) \left[\frac{1 + w_d}{1 + \frac{w_d}{2\tau}\sqrt{\left(\frac{p}{p_\alpha}\right)^3 \alpha}}\right]^3 n_d \qquad (47a)$$

In these formulas the quantities v, p, w_d and n_d refer to the small type, in this case a twin-float seaplane, and τ pertains to the enlarged edition.

As a matter of course, one may proceed by defining first the effect of the enlargement for the centric landing and then compute from the thus obtained impact $\bar{P}_d$, the impact $\bar{P}$ by one-sided landing.

In Wagner's formula for "V" bottoms, we again assume the impact during one-sided landing to be in the symmetrical plane of the float, so that (19), (41) and (42) yield

$$\bar{P} = c_2 \, v^2 \sqrt{a \, \bar{M}_r} = \sqrt{2\tau} \; P_d \tag{48}$$

For $\tau = 0.5$, we again have $\bar{P} = P_d$, and for the two practical limiting values of τ, we obtain:

$$\bar{P} = 0.72 \, P_d$$

and

$$\bar{P} = 1.35 \, P_d,$$

or an increase in impact up to 35%, independent of the size of the seaplane, over the hitherto prevailing specification.

Summary

The formula for the impact factor of floats must include the enlargement factor itself as well as the type of enlargement. The latter is preferably characterized by the change in surface loading. It is shown that the enlargement of a small seaplane generally results in a changed float (or boat) loading as well as wing loading.

The conditions of starting stipulate the retention of the float loading when changing from single-float (boat) to twin-float arrangement. This contingency is followed by an increased impact factor in the twin-float type against the otherwise equivalent single-float type.

The impact factor in a one-sided landing of a twin-float seaplane is dependent on the square of the ratio : eccentricity to inertia radius $\bar{r}^2/\bar{i}^2$, whose practical limits are 0.1 and 2.8. It is only when $\bar{r}^2/\bar{i}^2 = 1$, that the eccentric impact is half as high as the centric; only in this case do the specifications correspond to actual conditions. When $\frac{\bar{r}^2}{\bar{i}^2} > 1$ the eccentric impact is

smaller, when $\frac{\bar{r}^2}{i^2} < 1$, it is greater than half the centric impact.

These fundamental views are illustrated by application to Pabst's and Wagner's derived impact formulas which, as such, are not criticized. For evaluation two special types of enlargement are set up as practical limits: Lanchester's method, by which the surface loading is not disturbed, and Rohrbach's method, according to which the loading increases as the third root of the enlargement factor. The results of the inquiry confirm the theory fundamentally and lead to some practical conclusions in spite of the uncertainty of individual assumptions and of considerable discrepancies.

It is seen that Lanchester's method is preferable for the purpose of lower impact factor, and that the decrease in impact is much greater for "V" than for flat bottoms. A further result is that the ratio of impact factor for flat bottoms to that with a stated "V" shape, is different for different seaplane sizes.

Whereas the hitherto prevailing load specifications governing the impact of single and twin float types with flat bottoms conform to actual conditions in a certain measure, the respective stipulations, as applied to "V" bottoms, would have to be modified to the detriment of the twin-float or to the advantage of the single-float type.

The specifications governing the one-sided impact are also passably correct for flat bottoms, but at times seem to be too favorable; at other times, too unfavorable to "V" bottoms.

Instead of a general specification, the amount of the one-sided impact should preferably be defined from case to case by means of the seaplane mass reduced to the point of impact.

Translation by J. Vanier,
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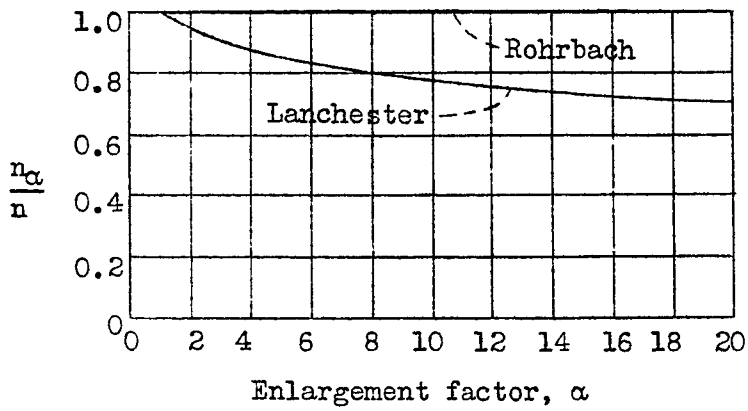

Fig. 1 Dependence of impact factor on the enlargement for flat bottoms. (Pabst formula)

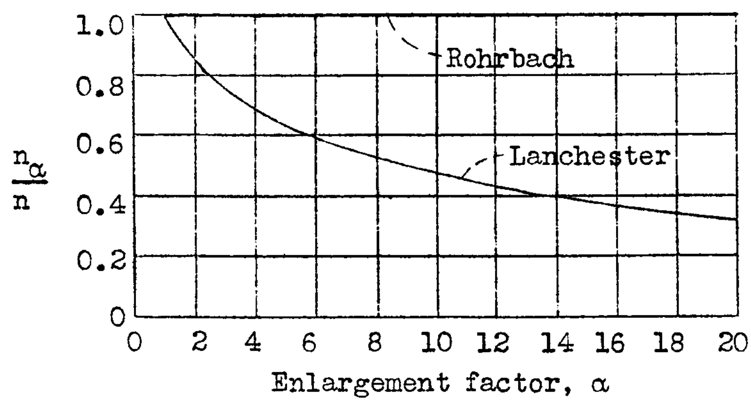

Fig. 2 Dependence of impact factor on the enlargement for "V" bottoms. (Pabst's formula)

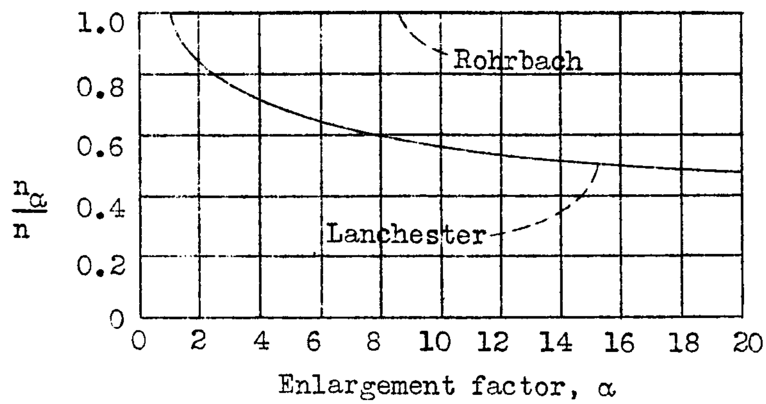

Fig. 3 Dependence of impact factor on the enlargement for " V " bottoms. (Wagner's formula)

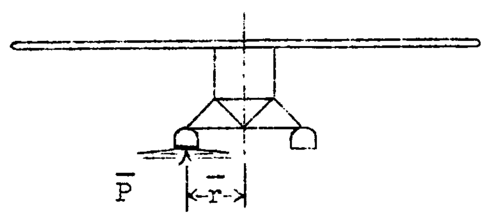

Fig. 4 One-sided landing of a seaplane.